Document 10.2.3.
9-fold Higher Order Patterning of *Tao Te Ching* Insights
Possibilities in the mathematics of magic squares, cubes and hypercubes

Introductory comments

The basis for Tables 1 and 2 of the 81 insights of the Tao Te Ching is discussed in a separate commentary, *Commentary on Tao Te Ching Interpretation and the possibility of higher order patterning*.

The rows of Table 1 provide 9 groups in terms of the conventional ordering in the Tao Te Ching. The columns of the table provide 9 different groups in terms of the alternative ordering represented by those columns.

Table 2 is an **experiment** in the organization of these insights into clusters. The table is made up of 9 nested tables (each of 9 cells). Each nested table corresponds to one of the rows from Table 1 – each row **above** being transformed into a nested table of 3x3 cells **below**. Note that the insight numbers in each row total to 369, as do the insight numbers in each column.

Magic squares

As a further experiment in organization, the insights were clustered according to the mathematical principle of the magic square (see Table 2). The structure of Table 2 is best understood by considering the first row of 9 insights (1 to 9) in Table 1. These 9 appear as the **central** nested table in the top row of 3 nested tables in Table 2. The 9 in that nested table are however presented in an order based on the structure of what is known in mathematics as a magic square – namely the numbers of the insights (of the conventional ordering in the Tao Te Ching), whatever the direction of addition, whether vertically (8+3+4; 1+5+9; 6+7+2), horizontally (8+1+6; 3+5+7; 4+9+2), or diagonally (8+5+2; 4+5+6), total in each case to 15 (as indicated there as 1:15). Similarly if the numbers of each row are multiplied (8x1x6; 3x5x7; 4x9x2) they together total to 225 – as do those of the columns (8x3x4; 1x5x9; 6x7x2).

In such a square the numbers of the first 9 insights (1 to 9) (of the conventional ordering in the Tao Te Ching), whatever the direction of addition, whether vertically (8+3+4; 1+5+9; 6+7+2), horizontally (8+1+6; 3+5+7; 4+9+2), or diagonally (8+5+2; 4+5+6), total in each case to 15 (as indicated there as 1:15). Similarly if the numbers of each row are multiplied (8x1x6; 3x5x7; 4x9x2) they together total to 225 – as do those of the columns (8x3x4; 1x5x9; 6x7x2).

This is an adaptation of the *Lo-Shu* order known in classical China. In the table as a whole, the 9 nested tables have been positioned in a manner corresponding to this **same** order.

Thus the first row of nested tables in Table 2 groups the contents of rows 8, 1 and 6 respectively from Table 1 (namely rows marked there as VIII, I, and VI), the second groups 3, 5 and 7, with the third grouping 4, 9 and 2. The principle of the magic square is discussed elsewhere (notably by Alan Grogono), together with its long history dating back to 2800 BC.

The Lo Shu is the only magic square of order 3. Namely there is just one 3x3 magic square – although with rotations and reflections, there are eight variations of what is essentially the same square. An associative magic square of order n is one for which every pair of numbers symmetrically opposite the center sum to n^2+1. The Lo Shu square is associative – but is not a panmagic square for which all the diagonals – including the broken diagonals obtained by "wrapping around" the edges – total like the rows and columns.

Just as the magic square total for the first 3x3 nested table is 15 (indicated in Table 2 as 1:15), each other 3x3 nested table gives rise to its own total (indicated beneath it, eg 4:96, 9:231, and 2:42). The 9 such totals from each nested table also constitute a magic square – with a total figure of 369. As might be expected, if the table as a whole is treated as a 9x9 magic square, the total is also 369.

Interesting patterns can be generated from magic squares when the numbers of the squares are replaced by symmetric symbols.

Pan-magic squares

Mathematically a "continuous" ("pan-magic", pan-diagonal, Nasik or Jaina) square has the additional property that even the broken diagonals add to the same total as those of the magic square. It was long supposed that a 9x9 pan-magic square did not exist, but one such based on the 81 numbers 0 to 80 is reported by Alan Grogono. He explains this early belief as probably due to the absence of any obvious pattern to use to create a regular 9x9 square. Constructing a square by expanding a 3x3 square indeed produces a magic square as in Table 2 but not a pan-magic one. In addition, amongst odd-order pan-magic squares, most interest has been focused on the regular prime number squares. These lent themselves to analysis more readily and to calculation of the number of regular pan-magic squares which could be constructed with an underlying pattern.

Grogono argues that the analysis (and construction) of magic squares is more logical, and the results make more sense, when the smallest number is 0 – instead of 1. This would imply that a 9x9 square of the Tao Te Ching insights should run from

Table 1

The rows of the table provide 9 groups in terms of the conventional ordering in the Tao Te Ching. The columns of the table provide 9 different groups in terms of the alternative ordering represented by those columns.

	a	b	c	d	e	f	g	h	i
I	1: Journeying through unnaming the myriad patterns of the past	2: Engaging without engaging	3: Cultivating non-engagement	4: Having been there; having done that	5: Engendering through complementarity	6: Completing	7: Enduring	8: Easing forward, going wherever, without competition	9: Avoiding excess
II	10: Centering through learning	11: Benefiting from what is not	12: Sensing the inner	13: Governing others appropriately	14: Living the present	15: Subtly stilling to clarify the troubled	16: Accepting wisely the enduring cycle of beginning and ending	17: Acting simply, enabling others to value their own initiatives	18: Failing to exalt merit
III	19: Being untroubled through needing little and wanting less	20: Living uncertainty, confusion and strangeness	21: Knowing the strange uncertainties offered in the moment	22: Acting contrarily	23: Being in the moment	24: Avoiding disproportion and self-satisfaction	25: Following the unnamable	26: Becoming insightful through assiduous handling of obligations	27: Educating the challenged as the inspiration of the wise
IV	28: Knowing the other, and retaining one's identity and quality	29: Doing "nothing" to the world	30: Leading through inspiration that does not seek to win	31: Using weapons, when there is no choice, with a calm, still mind	32: Knowing when to cease making essential distinctions	33: Applying to oneself the skills developed successfully to deal with others	34: Achieving greatness without great doings	35: Holding fast to the eternal process through the very ordinary	36: Prevailing through weakness
V	37: Self-organizing of myriad things	38: Abiding in letting go and doing nothing	39: Enwholing to sustain the integrity of the subtle	40: Returning from weakness	41: Understanding appropriateness	42: Losing as the key to the cycle of winning and losing	43: Ensubtling to enliven the impenetrable	44: Self-constraining fruitfully	45: Remaining calm and clear to ensure that the capacity for appropriateness is renewed
VI	46: Knowing that enough is enough	47: Understanding the truth and opportunity of the moment	48: Unlearning	49: Enminding the world to see the ordinary through the eyes of children	50: Living in recognition that this implies dying	51: Nurturing life according to natural processes	52: Understanding insignificant beginnings	53: Ensuring modesty	54: Ensuring that rules for oneself are consistent with those for the world
VII	55: Knowing harmony as knowing the eternal	56: Knowing that discourages talking	57: Avoiding instrumental thinking, initiation of innovative change and regulation of	58: Bumbling on without forcing	59: Gathering insight to ensure staying power	60: Allowing potentially disruptive forces to have their place	61: Lying low to ensure integrity and continuity	62: Honoring the appropriate as a gift	63: Focusing on the challenge of beginnings
VIII	64: Attending to what may have been neglected in the achievement of undertakings	65: Being in ignorance of appropriate action	66: Following rather than leading	67: Leading the mightiest by not presuming to do so	68: Avoiding competition	69: Yielding to antagonism	70: Being obscure	71: Knowing without knowing	72: Fearing the dangers of acting inappropriately
IX	73: Acting silently, non-competitively, and non-directively	74: Avoiding the presumptuousness of usurping the judgement on others	75: Living for more than the pursuit of wealth	76: Bending in response to pressure	77: Redistributing from those who have to those who have not	78: Recognizing the counter-intuitive, paradoxical nature of appropriate action	79: Fulfilling obligations	80: Enjoying the freedom of movement in relation to what is to hand	81: Doing without outdoing

Table 2

The table is made up of 9 nested tables (each of 9 cells). Each nested table corresponds to one of the rows from Table 1– each row **above** being transformed into a nested table of 3x3 cells **below**. Note that the insight numbers in each row total to 369, as do the insight numbers in each column.

71: Knowing without knowing	64: Attending to what may have been neglected in the achievement of undertakings	69: Yielding to antagonism	8: Easing forward, going wherever, without competition	1: Journeying through unnaming the myriad patterns of the past	6: Completing	53: Ensuring modesty	46: Knowing that enough is enough	51: Nurturing life according to natural processes
66: Following rather than leading	68: Avoiding competition	70: Being obscure	3: Cultivating non-engagement	5: Engendering through complementarity	7: Enduring	48: Unlearning	50: Living in recognition that this implies dying	52: Understanding insignificant beginnings
67: Leading the mightiest by not presuming to do so	72: Fearing the dangers of acting inappropriately	65: Being in ignorance of appropriate action	4: Having been there; having done that	9: Avoiding excess	2: Engaging without engaging	49: Enminding the world to see the ordinary through the eyes of children	54: Ensuring that rules for oneself are consistent with those for the world	47: Understanding the truth and opportunity of the moment
8:204			1:15			6:150		
26: Becoming insightful through assiduous handling of obligations	19: Being untroubled through needing little and wanting less	24: Avoiding disproportion and self-satisfaction	44: Self-constraining fruitfully	37: Self-organizing of myriad things	42: Losing as the key to the cycle of winning and losing	62: Honoring the appropriate as a gift	55: Knowing harmony as knowing the eternal	60: Allowing potentially disruptive forces to have their place
21: Knowing the strange uncertainties offered in the moment	23: Being in the moment	25: Following the unnamable	39: Enwholing to sustain the integrity of the subtle	41: Understanding appropriateness	43: Ensubtling to enliven the impenetrable	57: Avoiding instrumental thinking, initiation of innovative change and regulation of action	59: Gathering insight to ensure staying power	61: Lying low to ensure integrity and continuity
22: Acting contrarily	27: Educating the challenged as the inspiration of the wise	20: Living uncertainty, confusion and strangeness	40: Returning from weakness	45: Remaining calm and clear to ensure that the capacity for appropriateness is renewed	38: Abiding in letting go and doing nothing	58: Bumbling on without forcing	63: Focusing on the challenge of beginnings	56: Knowing that discourages talking
3:69			5:123			7:177		
35: Holding fast to the eternal process through the very ordinary	28: Knowing the other, and retaining one's identity and quality	33: Applying to oneself the skills developed successfully to deal with others	80: Enjoying the freedom of movement in relation to what is to hand	73: Acting silently, non-competitively, and non-directively	78: Recognizing the counter-intuitive, paradoxical nature of appropriate action	17: Acting simply, enabling others to value their own initiatives	10: Centering through learning	15: Subtly stilling to clarify the troubled
30: Lading through inspiration that does not seek to win	32: Knowing when to cease making essential distinctions	34: Achieving greatness without great doings	75: Living for more than the pursuit of wealth	77: Redistributing from those who have to those who have not	79: Fulfilling obligations	12: Sensing the inner	14: Living the present	16: Accepting wisely the enduring cycle of beginning and ending
31: Using weapons, when there is no choice, with a calm, still mind	36: Prevailing through weakness	29: Doing "nothing" to the world	76: Bending in response to pressure	81: Doing without outdoing	74: Avoiding the presumptuousness of usurping the judgement on others	13: Governing others appropriately	18: Failing to exalt merit	11: Benefiting from what is not
4:96			9:231			2:42		

0 to 80 instead of from 1 to 81. This would not affect the pattern of Table 2, provided that the rows from which it was derived in Table 1 were then renumbered from 0 to 8 (instead of from I to IX).

Of further interest, however, is to use the 9x9 pan-magic square order discovered by Grogono to redistribute the 81 insights. **There is an interesting clue to the relevance of renumbering the first insight from 1 to 0 – in the text of that first insight itself.**

Given the properties of the pan-magic square, in this case the row containing 0 (the insight traditionally numbered 1) in his case was shifted to the central position (and checked in the online facility he provides to ensure that it remained a pan-magic square). This gives Table 3a from which the ordering in Table 3b was then produced – **retaining the numbering of the insights in Table 1** (namely 0 in Table 3a is 1 in Table 3b, in order to correspond to Table 1).

In 1999 Dan Washburn made the point that "The vastu-purusha-mandala is a square of 81 subsquares with 9 subsquares on each side. Take a Lo Shu magic sqaure of 3 and place a Lo Shu magic square of 3 in each of its 9 subsquares and you have a 9 x 9 square of 81 subsquares. So the vastu-purusha-mandala is the Lo Shu square squared, or seen in more detail." According to Vini Nathan (*Vastu Purusha Mandala: Beyond Building Codes*, Nexus Network Journal, vol. 4, no. 3, Summer 2002), The Vastu purusha mandala has been defined as "a collection of rules which attempt to facilitate the translation of theological concepts into architectural form." This law of proportions and rhythmic ordering of elements not only found full expression in temples, but extended to residential and urban planning as well. He argues that the influence of the Vastu purusha mandala extended beyond building activity to encompass the cultural milieu as well.

In **Table 3a** it can be noted that the insight numbers in each row now total to 360 (instead of 369, as in Table 2), as do the insight numbers in each column.

In **Table 3b** it can be noted that the insight numbers in each row now total to 369 (as in Table 2, and in contrast to the 360 of Table 3a), as do the insight numbers in each column). In addition the total of the insight numbers in **any** 3x3 nested square (even across highlighting) also total to 369 – whereas those of the 3x3 nested squares (even those highlighted) in Table 2 are not equal (although those of the central 3x3 square **only** do indeed total to 369). Note that the difference of 9 between 360 and 369 derives from the difference in insight numbering from 0-80 against 1-81 (giving a difference of 9, whether in row or column totals).

Bimagic squares

Mathematically a magic square is bimagic (or 2-multimagic) if it remains "magic" after each of its numbers have been squared – a bimagic square thus has the additional property that if each number in the square is multiplied by itself (squared, or raised to the second power) the resulting row, column, and diagonal sums are also magic. Bimagic squares are a subset of the class of multimagic squares; it is believed that no bimagic squares of order less than 8 exists (Benson and Jacoby 1976). The original 3x3 Lo Shu square is far from being bimagic, since the sums of the squared numbers (of the rows or columns) vary between 77 and 107. The discoverer of the first bimagic square, G. Pfeffermann later published in *Les Tablettes du Chercheur* (15 July 1891) the first 9th-order bimagic square. In the case of the examples of bimagic squares based on 9x9 in Table 4, the rows and columns sum to 369 as before. But if each number is squared, the sum is then 20,049.

Most-perfect magic squares

A special type of pan-diagonal magic square is characterized as most-perfec. An example of a 12x12 most-perfect magic square is provided by Ian Stewart. The numbers in every 2x2 square sum to 286. More generally every 2 x 2 block of cells (including wrap-around) sum to 2T (where $T = n^2 + 1$). Any pair of integers distant ½n along a diagonal sum to T.

Table 3a
Distribution of 81 numbers according to 9x9 pan-magic pattern (as discovered by Alan Grogono)
Note that the insight numbers in each row now total to 360 (instead of 369, as in Table 2), as do the insight numbers in each column.

36	51	30	65	80	59	10	25	4
64	79	58	9	24	3	38	53	32
23	2	17	49	28	43	75	54	69
48	27	42	77	56	71	22	1	16
76	55	70	21	0	15	50	29	44
8	14	20	34	40	46	60	66	72
33	39	45	62	68	74	7	13	19
61	67	73	6	12	18	35	41	47
11	26	5	37	52	31	63	78	57

Table 3b: Application of pan-magic pattern to order 81 insights of Tao Te Ching

Note that the insight numbers in each row now total to 369 (as in Table 2, and in contrast to the 360 of Table 3a), as do the insight numbers in each column. In addition the total of the insight numbers in **any** 3x3 nested square (even across highlighting) also total to 369 – whereas those of the 3x3 nested squares (even those highlighted) in Table 2 are not equal (although those of the central 3x3 square **only** do indeed total to 369). Note that the difference of 9 between 360 and 369 derives from the difference in insight numbering from 0-80 against 1-81 (giving a difference of 9, whether in row or column totals).

37: Self-organizing of myriad things	52: Understanding insignificant beginnings	31: Using weapons, when there is no choice, with a calm, still mind	66: Following rather than leading	81: Doing without outdoing	60: Allowing potentially disruptive forces to have their place	11: Benefiting from what is not	26: Becoming insightful through assiduous handling of obligations	5: Engendering through complementarity
65: Being in ignorance of appropriate action	80: Enjoying the freedom of movement in relation to what is to hand	59: Gathering insight to ensure staying power	10: Centering through learning	25: Following the unnamable	4: Having been there; having done that	39: Enwholing to sustain the integrity of the subtle	54: Ensuring that rules for oneself are consistent with those for the world	33: Applying to oneself the skills developed successfully to deal with others
24: Avoiding disproportion and self-satisfaction	3: Cultivating non-engagement	18: Failing to exalt merit	50: Living in recognition that this implies dying	29: Doing "nothing" to the world	44: Self-constraining fruitfully	76: Bending in response to pressure	55: Knowing harmony as knowing the eternal	70: Being obscure
49: Enminding the world to see the ordinary through the eyes of children	28: Knowing the other, and retaining one's identity and quality	43: Ensubtling to enliven the impenetrable	78: Recognizing the counter-intuitive, paradoxical nature of appropriate action	57: Avoiding instrumental thinking, initiation of innovative change and regulation of	72: Fearing the dangers of acting inappropriately	23: Being in the moment	2: Engaging without engaging	17: Acting simply, enabling others to value their own initiatives
77: Redistributing from those who have to those who have not	56: Knowing that discourages talking	71: Knowing without knowing	22: Acting contrarily	1: Journeying through unnaming the myriad patterns of the past	16: Accepting wisely the enduring cycle of beginning and ending	51: Nurturing life according to natural processes	30: Leading through inspiration that does not seek to win	45: Remaining calm and clear to ensure that the capacity for appropriateness is renewed
9: Avoiding excess	15: Subtly stilling to clarify the troubled	21: Knowing the strange uncertainties offered in the moment	35: Holding fast to the eternal process through the very ordinary	41: Understanding appropriateness	47: Understanding the truth and opportunity of the moment	61: Lying low to ensure integrity and continuity	67: Leading the mightiest by not presuming to do so	73: Acting silently, non-competitively, and non-directively
34: Achieving greatness without great doings	40: Returning from weakness	46: Knowing that enough is enough	63: Focusing on the challenge of beginnings	69: Yielding to antagonism	75: Living for more than the pursuit of wealth	8: Easing forward, going wherever, without competition	14: Living the present	20: Living uncertainty, confusion and strangeness
62: Honoring the appropriate as a gift	68: Avoiding competition	74: Avoiding the presumptuousness of usurping the judgement on others	7: Enduring	13: Governing others appropriately	19: Being untroubled through needing little and wanting less	36: Prevailing through weakness	42: Losing as the key to the cycle of winning and losing	48: Unlearning
12: Sensing the inner	27: Educating the challenged as the inspiration of the wise	6: Completing	38: Abiding in letting go and doing nothing	53: Ensuring modesty	32: Knowing when to cease making essential distinctions	64: Attending to what may have been neglected in the achievement of undertakings	79: Fulfilling obligations	58: Bumbling on without forcing

Magic cubes

There are extensive resources on magic cubes and hypercubes [notably Harvey Heinz and Marián Trenkler] that may offer even more powerful ways of organizing the 81 insights. A magic cube is a three-dimensional version of the magic square in which the rows, columns, pillars (or "files"), and four space diagonals each sum to a single number known as the magic constant. If the cross section diagonals also sum to that constant, the magic cube is called a perfect magic cube; if they do not, the cube is called a semiperfect magic cube, or sometimes an Andrews cube (Gardner 1988). A pandiagonal cube is a perfect or semiperfect magic cube which is magic not only along the main space diagonals, but also on the broken space diagonals. In a panmagic square, in addition to the main diagonals, the broken diagonals also sum to the magic constant.

Harvey Heinz (*Magic Cubes - Introduction*, 2003) has reviewed the variety of, often confusing, definitions and features of "magic cubes" (see also his *Magic Cubes Definitions*, which includes a discussion of cube features) and has allocated them to distinct classes according to the types of parts that must sum correctly for the more advanced cubes. His classes may be summarized here as:

- **Simple:** Containing no, or less then 3m, orthogonal magic squares. Only the rows, columns, pillars and triagonals are required to sum correctly for a simple magic cube.
- **Pantriagonal:** All pantriagonals must sum correctly. There may be some simple and/or pandiagonal magic squares, but not enough to satisfy any other classifications.
- **Diagonal**: All 3m planar arrays must be 'simple' magic squares.
- **Pandiagonal:** All 3m planar arrays must be 'pandiagonal' magic squares. The 6 oblique squares are always magic. One of them may be pandiagonal magic.
- **Perfect:** All 3m planar arrays must be 'pandiagonal' magic squares. In addition, all pantriagonals must sum correctly. These two conditions combine to provide another 6m pandiagonal magic squares.

Heinz notes that a magic cube is called **normal** if it consists of the numbers 1 to m^3 (or 0 to $m^3 - 1$). A magic cube is called **associated** if all pairs of two numbers diametrically equidistant from the center of the cube equal the sum of the first and last number in the series. If the associated cube (or other dimension of hypercube) is an odd order, then the center of the cube is a cell containing one half the sum of the first and last number in the series.

Heinz provides a generalized definition as follows: A hypercube of dimension n is perfect if all pan-n-agonals sum correctly, and all lower dimension hypercubes contained in it are perfect! He also provides spreadsheets for testing them. Heinz has collaborated with J. R. Hendricks to produce a *A Unified Classification system for Magic Cubes* (Journal of Recreational Mathematics, 2002).

The relationship of the 81 tetragrams of the Taoist classic Tai Hsuan Ching (or Tài Xuán Jïng) and the Tao Te Ching has most recently been explored in relationship to modern physics by Tony Smith (*I Ching (Ho Tu and Lo Shu), Genetic Code, Tai Hsuan Ching, and the D4-D5-E6-E7-E8 VoDou Physics Model*). According to Smith:

> To construct the Tai Hsuan Ching, consider the Magic Square sequence as a line 3 8 4 9 5 1 6 2 7 with central 5 and opposite pairs at equal distances. If you try to make that, or a multiple of it, into a 9x9 Magic Square whose central number is the central number 41 of 9x9 = 81 = 40+1+40, you will fail because 41 is not a multiple of 5.

> However, since 365 = 5x73 is the central number of 729 = 364+1+364 , you can make a 9x9x9 Magic Cube with 9x9x9 = 729 entries, each 9x9 square of which is a Magic Square. The Magic Cube of the Tai Hsaun Ching gives the same sum for all lines parallel to an edge, and for all diagonals containing the central entry. The central number of the Magic Cube, 365....

> The total number for each line is 3,285 = 219 x 15. The total of all numbers is 266,085 = 5,913 x 45.

> Since 729 is the smallest odd number greater than 1 that is both a cubic number and a square number, the 729 entries of the 9x9x9 Magic Cube with central entry 365 can be rearranged to form a 27x27 Magic Square with 729 entries and central entry 365. 27 = 3x3x3 = 13+1+13 is a cubic number with central number 14, and there is a 3x3x3 Magic Cube with central entry 14 (14 is the dimension of the exceptional Lie algebra G2) and sum 42...

The I Ching is based on hexagrams of binary lines. Tony Smith, in his discussion of the Tai Hsuan Ching of ternary line tetragrams "arranged in T'ien" (as in Table 4d), the ternary numbers are given "plus 1", since the ternary numbers go from 0 to 80 (as indicated by Grogono above) instead of from 1 to 81.

This ternary number arrangement, according to Tony Smith, is similar to the *Fu Xi* binary number arrangement of the I Ching. This is not a magic square arrangement.

Magic hypercubes

A magic tesseract is a four-dimensional generalization of the two-dimensional magic square and the three-dimensional magic cube. Harvey Heinz defines a 4-dimensional hypercube (or tesseract) as perfect if all pan-quadragonals are correct, and all the magic squares and magic cubes within it are perfect. This means that the magic squares are all pandiagonal and the magic cubes are all pantriagonal and pandiagonal. There are $40m^2$ lines that sum correctly. They are m^3 rows, m^3 columns, m^3 pillars, m^3 files, $8m^3$ quadragonals, $16m^3$ triagonals, and $12m^3$ diagonals. Furthermore, a magic hypercube of any dimension n is perfect if all pan-n-agonals sum correctly, and all lower dimension hypercubes contained in it are perfect!

Table 4: **Magic squares from which bimagic squares can be generated**

Table 4a: from G. Pfeffermann: The first 9th-order bimagic square (*Les Tablettes du Chercheur,* 15 July 1891)

43	51	29	66	80	58	14	19	9
26	4	12	46	36	41	78	56	70
63	68	73	2	16	24	31	39	53
76	57	71	27	5	10	47	34	42
32	37	54	61	69	74	3	17	22
15	20	7	44	49	30	64	81	59
1	18	23	33	38	52	62	67	75
65	79	60	13	21	8	45	50	28
48	35	40	77	55	72	25	6	11

Table 4c: from David M. Collison (1991)

28	13	9	59	66	79	51	44	20
50	8	19	81	58	65	43	30	5
11	77	70	42	46	35	4	27	57
75	33	53	22	2	18	68	61	37
6	72	56	34	41	48	26	10	76
45	21	14	64	80	60	29	49	7
25	55	78	47	36	40	12	5	71
67	52	39	17	24	1	63	74	32
62	38	31	3	16	23	73	69	54

Table 4b: from J. R. Hendricks (Bimagic Squares: Order 9, Dec. 1999).

22	3	81	42	34	47	17	59	64
37	54	15	71	76	57	32	20	7
33	38	8	55	72	77	52	13	21
68	73	43	12	26	4	63	51	29
2	16	58	46	41	36	24	66	80
53	31	19	78	56	70	39	9	14
61	69	30	5	10	27	74	44	49
75	62	50	25	6	11	67	28	45
18	23	65	35	48	40	1	79	60

Table 4d: from Tony Smith (*I Ching (Ho Tu and Lo Shu), Genetic Code, Tai Hsuan Ching, and the D4-D5-E6-E7-E8 VoDou Physics Model*)

73	64	55	46	37	28	19	10	1
74	65	56	47	38	29	20	11	2
75	66	57	48	39	30	21	12	3
76	67	58	49	40	31	22	13	4
77	68	59	50	41	32	23	14	5
78	69	60	51	42	33	24	15	6
79	70	61	52	43	34	25	16	7
80	71	62	53	44	35	26	17	8
81	72	63	54	45	36	27	18	9